FAST COOLING OF NEUTRON STARS: SUPERFLUIDITY vs. HEATING AND ACCRETED ENVELOPE

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ABSTRACT

It is generally considered that the neutron star cooling scenarios involving fast neutrino emission, from a kaon or pion condensate, quark matter, or the direct Urca process, require the presence of baryon pairing in the central core of the star to control the strong neutrino emission and produce surface temperatures compatible with observations. I show here that within the kaon condensate scenario *pairing is not necessary* if: 1) the equation of state is stiff enough for the star to have a thick crust in which sufficient friction can occur to heat the star and 2) a thin layer, of mass ΔM larger than $\sim 10^{-12}\ M_\odot$, of light elements (H and He) is present at the stellar surface. The effect of the light elements is to increase the heat flow and thus produce a higher surface temperature. Both the occurrence of heating and the presence of H and/or He at the surface (deposited during the late post-supernova accretion) can possibly be confirmed or infirmed by future observations.

Subject headings: stars: neutron — dense matter — stars: X-rays — pulsars: general

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1. INTRODUCTION

There are indications that some young pulsars may have surface temperatures lower than what is predicted by the 'standard' model of neutron star cooling (see, e.g., Nomoto & Tsuruta 1986). Alternative models with fast neutrino emission (see, e.g., Page 1994), however, predict temperatures much lower than observed. It had been proposed (Page & Baron 1990; Page & Applegate 1992) that baryon pairing (i.e., neutron superfluidity and/or proton superconductivity, which strongly suppress the neutrino emission) in the densest regions of the neutron star core may be a way to keep the surface temperature high enough to be compatible with observations. Dissipation of the pulsar rotational energy into heat is another way to control the fast cooling, but detailed modeling of this effect (Shibazaki & Lamb 1988; Umeda et al 1993; Van Riper et al 1995) showed that it is by itself not sufficient. If the reported temperatures are close to the actual ones and if the actual temperatures are really due to the cooling of the neutron star then pairing at very high density seemed to be a mandatory ingredient for the viability of the fast cooling scenarios.

Recently, however, Potekhin, Chabrier & Yakovlev (1996) made an important point by noting that the presence of light elements (H and He) in the upper layers of the neutron star significantly affects the heat transport in these layers and results in a higher surface temperature when compared to the case of an iron envelope. They also showed that a layer of about $10^{-12}\ M_\odot$ of accreted matter (possibly deposited during the post-supernova accretion phase) is sufficient to lead to quite different predictions for cooling models. I will show here that this hypothesis, when applied to a cooling model with fast neutrino emission from a kaon condensate (i.e., not as fast as the direct Urca process, Thorsson et al 1995), and including a reasonable amount of internal heating, leads to temperatures much higher than previously calculated and that *'agreement with the data' can be obtained without the necessity of baryon pairing in the inner core where the fast neutrino emission is taking place*, a result in total opposition to the previous models mentioned above.

I summarize in § 2 the observational data on cooling neutron stars. My model is described in § 3 and the results are presented in § 4. The last section, 5, contains the discussion and conclusions.

2. OBSERVATIONAL DATA

I summarize in this section the information presently available about estimates of and upper limits on the effective temperature of cooling neutron stars and I list in table 1 all relevant data. There are several more objects which I omit because the deduced upper limits on T_e are still too high to be interesting.

The output of cooling models is a luminosity L^∞, i.e., an effective temperature T_e^∞ related to L^∞ by $L^\infty = 4\pi\sigma R^{\infty\,2}\, T_e^{\infty\,4}$ (σ being the Stefan-Boltzmann constant and R^∞ the neutron star's radius 'at infinity') and the observational problem is thus to deduce an accurate estimate of T_e^∞ from the data. The quantities 'at infinity' are related to the local ones through the redshift factor e^ϕ (= 0.78 for my 1.4 $M_\odot$ model) by: $L^\infty = e^{2\phi}L$, $T^\infty = e^\phi T$, and $R^\infty = e^{-\phi}R$. In the case of pulsars of goodness 1 to 3 (see table 1) the rough upper limits are based on a comparison with blackbody (BB) spectra. For the other five objects where there is some evidence in favor of the thermal origin of the detected photons (see, e.g., Ögelman 1995) the question of the spectrum used to obtain the temperature estimate is crucial. Model atmospheres (e.g., Romani 1987; Shibanov et al 1992) show that the BB spectrum may be a very poor approximation; in particular, magnetized hydrogen atmosphere spectra at high temperatures, when the atmosphere is wholly ionized, are much harder than BB spectra. At low temperature ($< 10^6$ K), however, bound-bound and bound-free absorption may soften substantially the magnetized hydrogen spectrum and push it toward or even below the BB spectrum (Pavlov & Potekhin 1995), but no accurate calculations of magnetized spectra have yet been published in this regime. In the case of the Vela pulsar, the BB fits give a high temperature, $T_{BB}^\infty \simeq 1.6\cdot 10^6$ K, but require a neutron star radius between $3-4$ km (Ögelman et al 1993) while fits with magnetized hydrogen atmosphere spectra, which are expected to be quite accurate at such high temperature, give a temperature $T_H^\infty \simeq 0.8\cdot 10^6$ K, corresponding to a surface temperature $T_H \simeq 10^6$ K, for a neutron star radius of 10 km, $R^\infty = 13.6$ km (Page et al 1996). However, if the luminosity deduced from the BB fits (with $R^\infty \sim 3-4$ km) is converted into an effective temperature T_e^∞ for a neutron star with $R^\infty = 13.6$ km, then $T_e^\infty \sim T_H^\infty$: this is the value I report in table 1. The neutron star 0002+6246 (Hailey & Craig 1995) is very similar to Vela and I calculate, in the same

way, T_e^∞ from T_{BB}^∞. In the other extreme, the cold Geminga, presently available magnetized atmosphere spectra are too hard, and give very low temperatures which require an enormously large star or, alternatively, a very small distance (Meyer et al 1993), while BB fits (Halpern & Ruderman 1993) give higher temperatures which are compatible with emission from the whole surface of the neutron star at the known distance of 157 (+59, -34) pc (Caraveo et al 1996): I thus choose to use T_{BB}^∞ in this case, i.e., I hope that $T_{BB}^\infty \sim T_e^\infty$. The two intermediate cases of PSR 0656+14 and PSR 1055-52 are more delicate: it is difficult to decide which of the BB or atmosphere spectra give temperature closer to the actual one. For PSR 0656+14, I report both values, T_{BB}^∞ (Finley et al 1992) and T_H^∞ (Anderson et al 1993), the actual value of T_e^∞ being probably somewhere in-between these, but for PSR 1055-52 only BB fits have been published (Ögelman & Finley 1993) so I use T_{BB}^∞ for T_e^∞.

3. PHYSICS OF THE MODEL

I will consider the fast neutrino cooling scenario induced by a kaon condensate (Brown et al 1988; Page & Baron 1990). The dense matter equation of state (EOS) is taken from Friedman & Pandharipande (1981), FP hereafter, and I assume that at densities above

$$\rho_{cr}^K = 1.1 \times 10^{15} \ \text{gm cm}^{-3} \simeq 4 \cdot \rho_0 \tag{1}$$

the kaon condensate develops and induces a fast neutrino emission

$$\epsilon_\nu^K = 10^{24} \ (\rho/\rho_0)^{2/3} \ T_9^6 \ \text{erg s}^{-1} \ \text{cm}^{-3} \tag{2}$$

(Thorsson et al 1995), where ρ is the matter density, $\rho_0 \equiv 2.8 \cdot 10^{14}$ gm cm^{-3} the nuclear matter density, and T_9 the temperature in units of 10^9 K. The critical neutron star mass for the appearance of a kaon condensate is then 1.24 $M_\odot$. The modified Urca process and the similar bremstrahlung processes are active at all densities within the core but, at a temperature of 10^8 K, the kaon induced processes are more efficient by five orders of magnitude. Of upmost importance for the cooling is the presence and extend of baryon pairing in the core: the corresponding critical temperatures T_c that I will use are plotted in figure 1. Notice that the assumed T_c's vanish within the condensate region. In regions where pairing is present, the specific heat of the paired component is strongly reduced as well as all neutrino emission processes to which this component participates.

Surface thermal emission is determined by the effective temperature T_e which is related to the interior temperature T_b through envelope models. T_b is traditionally taken at a density $\rho_b = 10^{10}$ gm cm^{-3}. The chemical composition of the envelope can strongly affect the $T_b - T_e$ relationship: compared to models with pure iron composition, models with accreted matter give a higher T_e for a given T_b. I will use the recent calculations of Potekhin et al (1996) for both pure iron envelopes and envelopes with a layer of accreted matter, mostly H and He, of mass ΔM. For a 1.4 $M_\odot$ star with the FP EOS, the total mass of the envelope, i.e., at densities below ρ_b, is about $3 \cdot 10^{-7}$ $M_\odot$. I neglect here the effects of the magnetic field which are small (Page & Sarmiento 1996).

Besides the neutrino and photon emissions which are energy sinks, rotational energy can be converted into heat by friction due to differential rotation of the inner crust neutron superfluid (Alpar et al 1984). I handle this effect in a simple way by writing the heating rate as

$$H(t) = J_{44} \cdot 10^{40} \cdot \left(\frac{t + \tau_0}{100 \text{ yrs}} \right)^{-3/2} \quad \text{erg s}^{-1} \tag{3}$$

where t is the pulsar age, $\tau_0 = 300$ yrs a typical spin-down time scale, and J_{44} the differential angular momentum of the frictionally coupled crustal neutron superfluid in units of 10^{44} g cm^2 rad s^{-1}. This expression assumes a standard spin-down rate from magnetic dipole radiation and is similar to the one used by Umeda et al (1993) and Shibazaki & Lamb (1988). This heating is distributed within the superfluid layers of the inner crust. I adopt the value $J_{44} = 3.1$ which corresponds to moderately strong heating, compatible with the size of the crust for the FP EOS.

My treatment of other physical ingredients is described in previous works (Page & Baron 1990; Page & Applegate 1992; Page 1994) and all calculations were performed with a wholly general relativistic Henyey-type evolutionary code.

4. RESULTS

Figure 2 shows results for the cooling of a 1.4 $M_\odot$ neutron star without and with internal heating and various amounts of accreted mass ΔM, as well as for a 1.2 $M_\odot$ star with heating and $\Delta M = 3 \cdot 10^{-8}$

$M_\odot$. As seen in figure 1, the 1.4 $M_\odot$ star contains a kaon condensate, of mass 0.3 $M_\odot$, while the 1.2 $M_\odot$ star has a central density below ρ_{cr} and thus follows the 'standard' cooling scenario. During the neutrino cooling era, i.e., at age up to about 10^4 – 10^5 yrs when the neutrino luminosity is much higher than the surface photon luminosity, the nature of the envelope material has no effect on the cooling rate but does affects the effective temperature: models with accreted envelope are warmer than models with iron envelope even though they all have the same interior temperature (considering separately models without heating and models with heating, the latter having of course a higher interior temperature than the formers). Even such a small amount of accreted mass as 10^{-12} $M_\odot$ has a very noticeable effect (Potekhin et al 1996). When photon cooling takes over, the cooling is accelerated in the cases of accreted envelopes because of the increased surface emission. Finally, notice that at the latest ages the cooling is entirely controlled by the heating, independently of the previous history or envelope composition: the photon luminosity is simply equal to the heating rate.

Comparing these cooling curves, when both heating and an accreted envelope are taken into account, with the plotted surface temperatures, one sees that the 'standard' cooling scenario, i.e., M < 1.24 $M_\odot$ in my model, seems to be appropriate for PSR 1055-52 and maybe also PSR 0656+14 if the higher temperature estimate is considered for the latter. The temperature 'measurements' of Vela, NS 0002+6246, Geminga, and PSR 0656+14 if the lower T_e value of the latter is considered, are below the standard cooling curve but can be fitted very well with the kaon condensate scenario in presence of heating and with an accreted envelope of mass larger than $\sim 3 \cdot 10^{-12}$ $M_\odot$.

5. DISCUSSION AND CONCLUSIONS

The results of the previous section show that agreement between cooling models with fast neutrino emission and the best presently available data does not necessarily require the presence of baryon pairing in the inner core of the neutron star: the superfluidity T_c's assumed in the calculations (figure 1) vanish within the kaon condensate region. The three basic ingredients needed to obtain temperatures 'in agreement with the data' are: 1) fast neutrino emission as supplied by a kaon condensate, 2) moderately strong internal heating and 3) the presence of light elements in the envelope.

There is no consensus on which, if any at all, fast neutrino emission mechanism is possible in neutron star cores but the kaon condensate is the least efficient of all: a pion condensate or the direct Urca process (Thorsson et al 1995) would shift downward the cooling curves during the neutrino cooling era and require much higher heating rates.

The internal heating by friction of the inner crust differentially rotating neutron superfluid depends on poorly known microscopic parameters (see, e.g., Van Riper et al 1995). A much stronger heating rate than assumed here is theoretically possible but would conflict with the temperature upper limits of PSR 1929+10 and 0950+08 (see figure 2 and Van Riper et al 1995). Efficient heating, moreover, requires a thick enough crust, i.e., a rather stiff EOS, and there are arguments for such an EOS precisely in the case of the Vela pulsar from the analysis of its glitches (Link et al 1992) and from the effect of gravitational lensing on the amplitude of the pulses seen in its surface thermal emission (Page & Sarmiento 1996). An extreme softening of the EOS in presence of kaon condensation was the result of the first models (Thorsson et al 1994) but the latest results of Fujii et al (1996) show that relativistic effects strongly reduce the softening of the EOS by the condensate: a stiff EOS even in the presence of a kaon condensate is thus not unlikely. Finally, in presence of heating the independence of the late cooling on the previous history and envelope structure could allow a determination of the microscopic parameters involved in the problem when detection of thermal emission from old pulsars will be achieved (Van Riper et al 1995). Actually, recent Hubble observations (Pavlov et al 1996a) may have detected the optical – UV Rayleigh-Jeans tail of the surface thermal emission of PSR 1929+10 and 0950+08 and indicate surface temperatures of the order of $1 - 3 \cdot 10^5$ K for the former and $6 - 8 \cdot 10^4$ K for the latter. This estimated low temperature of PSR 0950+08 is much below our simple model prediction but well above any cooling curve which does not include any heating mechanism: if confirmed, it would show that *heating is required* but that the simplistic formula 3 is obviously much too naive.

Due to the enormous uncertainty about the value of the pairing critical temperature T_c at high density (see, e.g., Page 1994 for a discussion) it is a relief that the fast cooling scenarios, preferentially the kaon con-

densate one, do not necessarily require the presence of superfluidity. Takatsuka & Tamagaki (1995) recently emphasized that in presence of a kaon condensate both neutron and protons would pair in a $^3P_2$ state, because of the high proton fraction, and nucleon pairing is then unlikely unless the neutron, or proton, effective mass is unrealistically high. The relativistic effects described by Fujii et al (1996) imply precisely a strong reduction of the nucleon effective mass within the condensate region and disfavor pairing. However these effects also reduce significantly the softening of the EOS, implying thicker crust and hence higher heating rate, and the low nucleon effective masses also reduce ϵ_ν^K (Brown et al 1988).

Finally, the main question arisen by the results presented above is the possible presence of light elements at the surface of a neutron star. The late post-supernova accretion could deposit the needed 10^{-12} $M_\odot$ of light elements. The only indication I know of about the presence of hydrogen is in the case of the Vela pulsar when comparing spectral fits with BB spectra vs. magnetized hydrogen atmosphere spectra as discussed in § 2 which clearly show the inadequacy of blackbody-like spectra. Magnetized iron atmosphere spectra (Rajagopal et al 1996) are blackbody-like when used to fit ROSAT's PSPC low energy resolution spectra: they would imply, for Vela, temperatures and stellar radii similar to the BB fits, i.e., radii at infinity R^∞ of the order of $3-4$ km compared to 14 km for magnetized hydrogen fits. Multiwevelength studies (Pavlov et al 1996b) and the future X-ray missions like SRG, AXAF, and XMM, will help resolve this issue.

I have thus replaced the 'magic word' *superfluidity* by *heating and accreted envelope*: apparently one of them is needed to make the fast cooling scenarios viable. The presence of heating and/or of accreted matter will be confronted by observations in the near future while superfluidity has probably little chance of getting out of the theoretical realm.

As a last comment I must emphasize that the 'classification' of the observed neutron stars as following the slow ('standard') cooling scenario or some fast cooling scenario is model dependent. For example, the present results imply that Geminga and PSR 0656+14 (if the lower T_e is used) belong to the fast cooling scenario but different assumptions about the extend of baryon pairing in the core can accommodate them within the 'standard' cooling scenario (Page 1994). Moreover, Vela and PSR 0002+6246 could also be accommodated within the 'standard' cooling scenario with a radically different assumption about the efficiency of the modified Urca rate (Schaab et al 1996).

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Fig. 1.— Critical temperatures T_c for neutron and proton pairing in the $^1S_0$ and $^3P_2$ channels used in this work (Page 1994). Indicated are also the central densities of the 1.2 $M_\odot$ and 1.4 $M_\odot$ neutron star models used and the crust-core and kaon condensate boundaries.

Fig. 2.— Thin dotted line: Cooling curves for a 1.2 $M_\odot$ neutron star ('standard' cooling) with heating and accreted envelope ($\Delta M = 3 \cdot 10^{-8}$). Thick lines: cooling curves for a 1.4 $M_\odot$ neutron star with a kaon condensate, various accreted masses and with or without heating. The data are from table 1.

Table 1
Observational data on neutron star surface temperature [a]

NS	Log Age (yrs)	Distance (kpc)	Log T_e^∞ (K)	Goodness [b]	Reference
0531+21	3.10	2.	< 6.25	3	Becker & Ashenbach 1995
0833-45	4.05	0.5	5.87 – 5.91	5	Page et al 1996
"	"	"	5.81 – 5.94	5	Ögelman et al 1993 [c]
0002+6246	~ 4	~ 3	5.72 – 5.85	4	Hailey & Craig 1995 [c]
1706-44	4.24	1.8	< 6.1	2 - 3	Becker et al 1995
2334+61	4.61	2.5	< 6.1	2	Slane & Lloyd 1995
1916+14	4.95	1.5	< 6.0	1	Slane & Lloyd 1995
0656+14	5.04	0.5	5.93 – 5.97	5	Finley et al 1992
"	"	0.3	5.70 – 5.76	"	Anderson et al 1993
0740-28	5.20	1.9	< 6.0	1	Slane & Lloyd 1995
1822-09	5.37	1.0	< 5.85	1	Slane & Lloyd 1995
0630+178	5.48	0.16	5.60 – 5.70	5	Halpern & Ruderman 1993
1055-52	5.73	0.5	5.74 – 5.82	5	Ögelman & Finley 1993
1929+10	6.49	0.17	< 5.5	3	Yancopoulos et al 1993
0950+08	7.23	0.13	< 5.1	2	Manning & Wilmore 1994

[a]Listed are: neutron star name; log of age; distance; log of effective temperature; 'goodness' [b]of the data; reference.

[b]Data 'goodness': 1: neutron star not detected, 2: neutron star detected at low count rate which precludes any serious analysis of the origin of the photons, 3: neutron star clearly detected but with evidence that the photons come mostly, or even exclusively, *not* from surface thermal emission, 4: neutron star clearly detected with some spectral evidence about the thermal origin of the photons and, finally, 5: neutron star clearly detected with good spectral evidence about the thermal origin of the photons.

[c]The T_e^∞ reported here is really an *effective temperature* obtained from the luminosity as discussed in section 2 and not the blackbody temperature T_{BB}^∞ reported by the authors.

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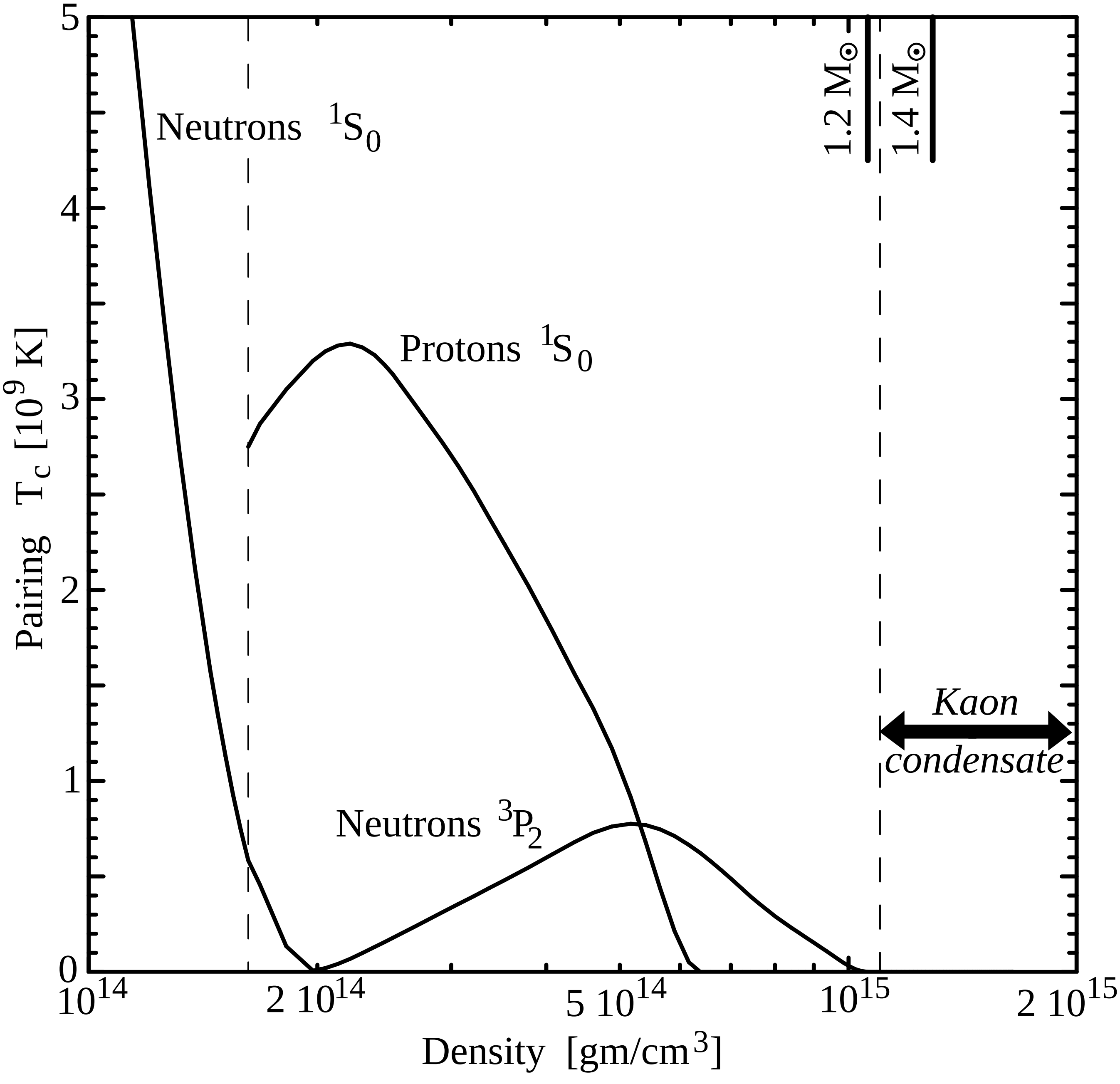

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Figure 1

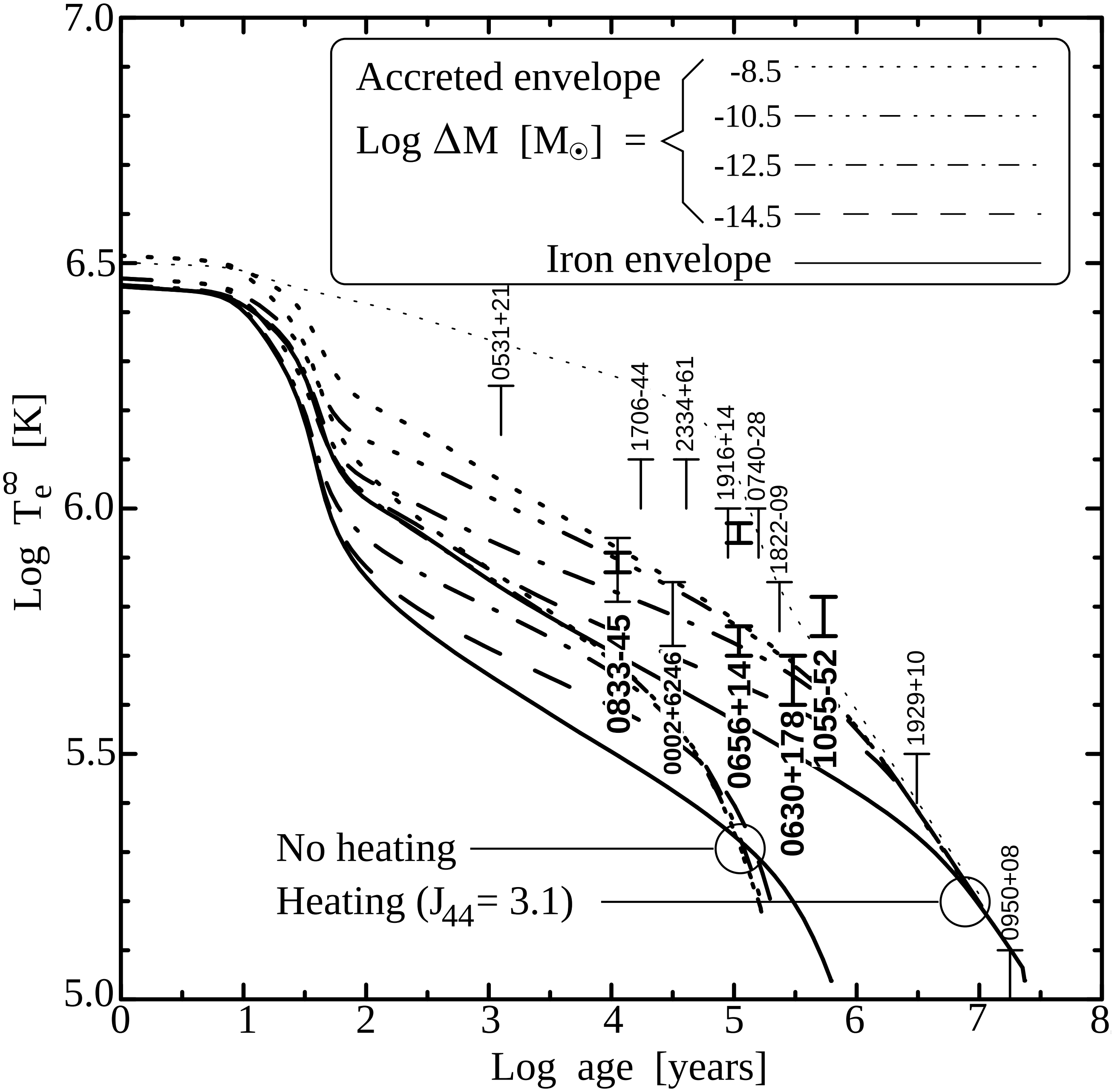

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Figure 2